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Gravitational, Electromagnetic, and Nuclear Theory

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Through an examination of neutron lifetimes we shall propose a new theory of gravitational, electromagnetic, and nuclear fields. The fundamental assumption of our theory is that the motion of a particle in a combination of gravitational, electromagnetic, and nuclear fields is determined from a variational principle of the form $\delta \int_A^B d\tau = 0$. The form of the physical time is determined from an examination of classical equations of motion. The field equations are determined from an examination of Maxwell–Einstein theory and Yukawa theory. Utilizing the standard elementary model of the deuteron, the theory predicts that at a proton–neutron separation $r \sim 10^{-13}$ cm the neutron lifetime is infinite and that nucleons have a repulsive core. These predictions have been experimentally verified.

1. INTRODUCTION

This paper is a continuation of a previous paper (Apsel, 1979) in which we introduced a new theory of gravitation and electromagnetism. Our previous investigation led to the prediction that physical time can be altered by electromagnetic potentials along with gravitational potentials. A recent measurement of muon lifetimes in the presence of strong electromagnetic fields performed at CERN by J. Bailey et al. (1977)[1] indicates that this prediction is correct.

In free space the neutron has a lifetime of about 10^3 sec. However, in a deuteron the neutron lifetime is infinite. This suggests that nuclear potentials, like gravitational and electromagnetic potentials, can alter physical time.

In Section 2 we shall propose a new formalism which will satisfy the

[1] This experiment and its relation to the prediction that electromagnetic potentials can alter the lifetime of a particle are discussed in Apsel (1979).

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conditions imposed by the observed alteration in neutron lifetimes. In Section 3 we apply the theory to an investigation of the deuteron. We make the simplest possible assumption: that the nuclear potential acting on the neutron in the deuteron depends only upon the proton–neutron separation. This is the standard elementary model of the deuteron first investigated by Wigner (1932). We find that the radius of the ground state deuteron is $r \sim 10^{-13}$ cm, and that at a proton–neutron separation of r the neutron lifetime is infinite. We also find that for a proton–neutron separation less than r the "force" acting on each nucleon becomes strongly repulsive. Thus each nucleon has a "repulsive core." The existence of a "repulsive core" was first proposed by Heisenberg (1933) and first observed by Jastrow (1951).

2. EQUATIONS OF MOTION AND FIELD EQUATIONS

From our previous paper (Apsel, 1979), we have that space-time is a Riemannian space with metric $g_{\mu\nu}$, and that the law of motion for a particle in a combination of gravitational and electromagnetic fields is $\delta \int_A^B d\tau = 0$, where

$$d\tau = \frac{1}{c}\left[(g_{\mu\nu}\, dx^\mu\, dx^\nu)^{1/2} + \frac{q}{mc^2} A_\mu\, dx^\mu\right]$$

It is natural to assume that $\delta \int_A^B d\tau = 0$ is the correct law of motion for a particle in a combination of gravitational, electromagnetic, and nuclear fields. This implies that $d\tau$ must be further generalized.

Our primary consideration is that our equations of motion reduce to the usual classical equations of motion for a particle in a classical nuclear field. A natural choice for $d\tau$ which will achieve this goal is

$$d\tau = \frac{1}{c}\left[\left(1 + \frac{f\phi}{mc^2}\right)(g_{\mu\nu}\, dx^\mu\, dx^\nu)^{1/2} + \frac{q}{mc^2} A_\mu\, dx^\mu\right]$$

where f is the "mesonic charge" (see Born, 1962) and ϕ is a scalar which represents the nuclear potential. Inserting $d\tau$ into $\delta \int_A^B d\tau = 0$ our equations of motion become

$$\frac{du^\mu}{ds} + \left\{ {\mu \atop \nu\sigma} \right\} u^\nu u^\sigma + \frac{f}{mc^2}\frac{g^{\mu\gamma}}{(1 + f\phi/mc^2)}(\phi_{,\sigma} g_{\gamma\nu} - \phi_{,\gamma} g_{\sigma\nu}) u^\nu u^\sigma$$
$$- \frac{q}{mc^2}\frac{1}{(1 + f\phi/mc^2)} F^\mu{}_\nu u^\nu = 0$$

where $ds = (g_{\mu\nu}\,dx^\mu\,dx^\nu)^{1/2}$ and $u^\mu = dx^\mu/ds$. Or, using the parameter λ, where

$$\lambda(A, B) \equiv \int_A^B \left(1 + \frac{f\phi}{mc^2}\right)(g_{\mu\nu}\,dx^\mu\,dx^\nu)^{1/2}$$

and thus

$$\frac{d\lambda}{ds} = \left(1 + \frac{f\phi}{mc^2}\right)$$

we have

$$\frac{d\omega^\mu}{d\lambda} + \left\{ {\mu \atop \nu\sigma} \right\}\omega^\nu\omega^\sigma + \frac{f}{mc^2}\frac{g^{\mu\gamma}}{(1 + f\phi/mc^2)}(\phi_{,\sigma}g_{\gamma\nu} + \phi_{,\nu}g_{\sigma\gamma} - \phi_{,\gamma}g_{\nu\sigma})\omega^\nu\omega^\sigma - \frac{q}{mc^2}\frac{1}{(1 + f\phi/mc^2)}F^\mu{}_\nu\omega^\nu = 0$$

where $\omega^\mu = dx^\mu/d\lambda$.

If we consider a system in which $g_{\mu\nu} = \epsilon_{\mu\nu}$, $A_\mu = 0$, and $v \ll c$ our equations of motion become

$$\frac{d^2x^i}{dt^2} = -\frac{f}{m}\frac{1}{(1 + f\phi/mc^2)}\delta^{ij}\frac{\partial\phi}{\partial x^j}, \qquad i, j = 1, 2, 3$$

For $f\phi \ll mc^2$ this is Newton's law of motion for a nuclear potential.

Light signals are assumed to travel along null lines. In our previous paper we showed that $g_{\mu\nu}$ and A_μ are measurable. If we measure $d\tau$ of a nucleon in a known $g_{\mu\nu}$ and A_μ field we can determine ϕ. Thus ϕ is measurable.

Field equations cannot be chosen independently of equations of motion. Therefore we must choose field equations that both reduce to accepted equations and yield the proper internal consistency. From our previous paper the gravitational and electromagnetic field equations are

$$G_{\mu\nu} = -\frac{8\pi k}{c^4}T_{\mu\nu}, \qquad F^{\mu\nu}{}_{;\nu} = \frac{4\pi}{c}J^\mu, \qquad A^\mu{}_{;\mu} = 0$$

For $g_{\mu\nu} = \epsilon_{\mu\nu}$, the Yukawa potential (Yukawa, 1935) satisfies the equations

$$\epsilon^{\mu\nu}\phi_{,\mu\nu} + \frac{m_0{}^2c^2}{\hbar^2}\phi = -4\pi P$$

where m_0 is the π-meson mass, $\hbar$ is Planck's constant divided by 2π, ϕ is the Yukawa potential, and P is the source term. The success of Yukawa's theory suggests that we choose this as our field equation for ϕ. For any $g_{\mu\nu}$ we have

$$g^{\mu\nu}\phi_{;\mu\nu} + \frac{m_0{}^2c^2}{\hbar^2}\phi = -4\pi P$$

For a system of N point particles let us define the scalar densities

$$\rho = \frac{1}{(-g)^{1/2}}\sum_A m_A\,\delta\left(x^i - \underset{A}{\xi^i}\right)\frac{ds}{dx^4}$$

$$\rho' = \frac{1}{(-g)^{1/2}}\sum_A q_A\,\delta\left(x^i - \underset{A}{\xi^i}\right)\frac{ds}{dx^4}$$

$$\rho'' = \frac{1}{(-g)^{1/2}}\sum_A f_A\,\delta\left(x^i - \underset{A}{\xi^i}\right)\frac{ds}{dx^4}$$

where $A = 1, 2, \ldots, N$, δ is the three-dimensional delta function, and $\underset{A}{\xi^i}$ is the position of Ath particle. The expression for J^μ in terms of source parameters takes the usual form

$$J^\mu = \rho' c u^\mu$$

According to Yukawa's theory, the expression for P should be

$$P = \rho''$$

We shall now find an expression for $T^{\mu\nu}$ by using the standard technique of determining $T^{\mu\nu}$ for $g_{\mu\nu} = \epsilon_{\mu\nu}$ and then generalizing to arbitrary $g_{\mu\nu}$. For $g_{\mu\nu} = \epsilon_{\mu\nu}$ we can write our equations of motion in the form

$$\frac{d\lambda}{ds}\rho c^2\frac{du^\mu}{ds} + \rho''\epsilon^{\mu\gamma}(\phi_{,\nu}\epsilon_{\gamma\sigma} - \phi_{,\gamma}\epsilon_{\sigma\nu})u^\sigma u^\nu - \frac{1}{c}F^{\mu\nu}J_\nu = 0$$

where

$$\frac{d\lambda}{ds} = \left(1 + \frac{f_A\phi}{m_A c^2}\right)$$

along with Ath trajectory. The left-hand side of this equation is a natural choice for $T^{\mu\nu}{}_{,\nu}$.[2] By conservation of mass $(\rho c u^\nu)_{,\nu} = 0$, and consequently

$$\frac{d\lambda}{ds}\left(\rho c^2\frac{du^\mu}{ds}\right) = \frac{d\lambda}{ds}(\rho c^2 u^\mu u^\nu)_{,\nu}$$

[2] Another possibility is to define

$$T^{\mu\nu}{}_{,\nu} = \rho c^2\frac{du^\mu}{ds} + \frac{ds}{d\lambda}\rho''\epsilon^{\mu\gamma}(\phi_{,\nu}\epsilon_{\gamma\sigma} - \phi_{,\gamma}\epsilon_{\sigma\nu})u^\sigma u^\nu - \frac{1}{c}\frac{ds}{d\lambda}F^{\mu\nu}J_\nu$$

Along the Ath trajectory $(m_A/f_A)\rho'' = \rho$, and consequently

$$\rho''\epsilon^{\mu\gamma}\phi_{,\nu}\epsilon_{\gamma\sigma}u^\sigma u^\nu = \left(\frac{d\lambda}{ds}\right)_{,\nu}\rho c^2 u^\mu u^\nu$$

It now follows that

$$T^{\mu\nu}{}_{,\nu} = \left(\frac{d\lambda}{ds}\rho c^2 u^\mu u^\nu\right)_{,\nu} - \rho''\epsilon^{\mu\nu}\phi_{,\nu} - \frac{1}{c}F^{\mu\nu}J_\nu$$

We have

$$-\rho''\epsilon^{\mu\nu}\phi_{,\nu} = \frac{1}{4\pi}\left[\epsilon^{\mu\sigma}\epsilon^{\nu\gamma}\phi_{,\sigma\nu}\phi_{,\gamma} - \tfrac{1}{2}\epsilon^{\mu\nu}\epsilon^{\sigma\gamma}(\phi_{,\sigma\nu}\phi_{,\gamma} + \phi_{,\sigma}\phi_{,\gamma\nu})\right]$$

$$+ \frac{1}{4\pi}\left[\left(\epsilon^{\sigma\gamma}\phi_{,\sigma\gamma} + \frac{m_0{}^2c^2}{\hbar^2}\phi\right)\epsilon^{\mu\nu}\phi_{,\nu}\right]$$

where the term in the first square bracket can be shown to vanish by a rearrangement of dummy suffixes, and the remaining term on the right-hand side can be shown to equal the left-hand side by use of the field equation for ϕ. Rearranging terms we have

$$-\rho''\epsilon^{\mu\nu}\phi_{,\nu} = \frac{1}{4\pi}\left[(\epsilon^{\mu\sigma}\epsilon^{\nu\gamma} - \tfrac{1}{2}\epsilon^{\mu\nu}\epsilon^{\sigma\gamma})\phi_{,\sigma}\phi_{,\gamma} + \tfrac{1}{2}\frac{m_0{}^2c^2}{\hbar^2}\phi^2\epsilon^{\mu\nu}\right]_{,\nu}$$

We can show in the usual way that

$$-\frac{1}{c}F^{\mu\nu}J_\nu = \frac{1}{4\pi}(-F^\mu{}_\sigma F^{\nu\sigma} + \tfrac{1}{4}F^{\sigma\gamma}F_{\sigma\gamma}\epsilon^{\mu\nu})_{,\nu}$$

and consequently we can integrate $T^{\mu\nu}$

$$T^{\mu\nu} = \frac{d\lambda}{ds}\rho c^2 u^\mu u^\nu + \frac{1}{4\pi}\left[(\epsilon^{\mu\sigma}\epsilon^{\nu\gamma} - \tfrac{1}{2}\epsilon^{\mu\nu}\epsilon^{\sigma\gamma})\phi_{,\sigma}\phi_{,\gamma} + \tfrac{1}{2}\frac{m_0{}^2c^2}{\hbar^2}\phi^2\epsilon^{\mu\nu}\right]$$

$$+ \frac{1}{4\pi}(-F^\mu{}_\sigma F^{\nu\sigma} + \tfrac{1}{4}F^{\sigma\gamma}F_{\sigma\gamma}\epsilon^{\mu\nu})$$

For arbitrary $g_{\mu\nu}$

$$T^{\mu\nu} = \frac{d\lambda}{ds}\rho c^2 u^\mu u^\nu + \frac{1}{4\pi}\left[(g^{\mu\sigma}g^{\nu\gamma} - \tfrac{1}{2}g^{\mu\nu}g^{\sigma\gamma})\phi_{;\sigma}\phi_{;\gamma} + \tfrac{1}{2}\frac{m_0{}^2c^2}{\hbar^2}\phi^2 g^{\mu\nu}\right]$$

$$+ \frac{1}{4\pi}(-F^\mu{}_\sigma F^{\nu\sigma} + \tfrac{1}{4}F^{\sigma\gamma}F_{\sigma\gamma}g^{\mu\nu})$$

Using the correspondence principle it follows immediately that for $v \ll c$, the quantized version of our theory is formally identical to the usual nonrelativistic quantum nuclear theory.

3. THE DEUTERON

The deuteron is composed of a proton and neutron. Consider a coordinate system in which $g_{\mu\nu} = \epsilon_{\mu\nu}$. For the neutron we have

$$d\tau = \frac{1}{C}\left(1 + \frac{f\phi}{mc^2}\right)(\epsilon_{\mu\nu}\, dx^\mu\, dx^\nu)^{1/2}$$

where f and m are the mesonic charge and the mass of the neutron. We have $d\tau = 0$ when

$$1 + \frac{f\phi}{mc^2} = 0$$

We make the simplest possible assumption: that the nuclear potential acting on the neutron depends only upon the separation between the neutron and proton. This is the standard elementary model of the deuteron first investigated by Wigner (1932). It follows from our field equations that ϕ is of the form

$$\phi = -f\frac{\exp\left[-(m_0 c/\hbar)r\right]}{r}$$

Inserting this into our condition for $d\tau = 0$ we have

$$-\phi = f\frac{\exp\left[-r/r_0\right]}{r} = \frac{mc^2}{f}, \qquad r_0 \equiv \frac{\hbar}{m_0 c}$$

$$\frac{r}{r_0}\exp\left[\frac{r}{r_0}\right] = k, \qquad k \equiv \frac{m_0 f^2}{mc\hbar}$$

Using the well-known values $m/m_0 \simeq 7.0$ and $f^2 \simeq 15\hbar c$, we have $k \simeq 15/7$ and

$$r \simeq 0.9 r_0 \simeq 1.3 \times 10^{-13}\ \text{cm}$$

For the neutron, $(\epsilon_{\mu\nu}\, dx^\mu\, dx^\nu)^{1/2} = (c^2 - v^2)^{1/2}\, dt$ is positive. Therefore, a proton–neutron separation of less than r would yield a negative physical time increment for the neutron, a result which we exclude as a possibility. Thus r is the minimum proton–neutron separation; that is, the radius of the ground state deuteron is r. In the ground state deuteron the physical time increments of the neutron are zero, and consequently the neutron lifetime is infinite.

Assuming $v \ll c$, for $q = 0$ or $A_\mu = 0$ we have shown the equations of motion to be

$$\frac{d^2x^i}{dt^2} = -\frac{f}{m}\frac{1}{(1 + f\phi/mc^2)}\,\delta^{ij}\,\frac{\partial\phi}{\partial x^j}$$

from which it follows that for the proton–neutron separation to become smaller than r each nucleon must pass an infinite repulsive force. Thus each nucleon has a "repulsive core." The existence of a "repulsive core" was first proposed by Heisenberg (1933) and first observed by Jastrow (1951).

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